The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Final Exam

Date: December 18, 2017 Course: EE 313 Evans

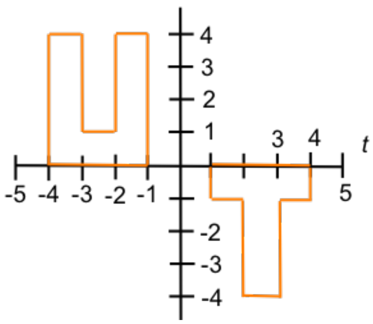
Name: ____
Last, First

- The exam is scheduled to last three hours.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- ***Please disable all wireless connections on your calculator(s) and computer system(s).***
- Please turn off all cell phones.
- No headphones are allowed.
- All work should be performed on the midterm exam. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

Problem	Point Value	Your score	Topic
1	10		Continuous-Time Fourier Series
2	12		Discrete-Time Convolution
3	9		Discrete-Time Convolution II
4	12		Continuous-Time Convolution
5	12		Discrete-Time FIR Filter Design
6	12		Discrete-Time IIR Filter Design
7	12		Continuous-Time Feedback System
8	9		Continuous-Time Circuit Analysis
9	12		Sinusoidal Amplitude Modulation
Total	100		

Problem 1. Continuous-Time Fourier Series. *10 points*.

One period of a periodic "UT" signal is shown below:



The fundamental period T_0 is 10 seconds.

(a) Compute the Fourier series coefficients. *9 points*.

(b) If the periodic "UT" signal is synthesized using 100 Fourier series coefficients, will it suffer from Gibbs phenomenon? *1 point*.

Problem 2. Discrete-Time Convolution. *12 points*.

Using forward and inverse z-transforms, derive the formula in the time domain for

$$y[n] = x[n] * h[n]$$

where

$$x[n] = na^n u[n] \text{ and } h[n] = b^n u[n]$$

Here, a and b are complex-valued constants such that $a \neq b$.

Problem 3. Discrete-Time Convolution II. *9 points*.

Compute the discrete-time convolution

$$y[n] = x[n] * h[n]$$

where

$x[n]$ is a causal rectangular pulse with an amplitude of 1 and a duration of N_x samples, and

$h[n]$ is a causal rectangular pulse with an amplitude of 1 and a duration of N_h samples.

(a) Give a formula for $y[n]$ in terms of N_x and N_h. *6 points.*

(b) Plot $y[n]$. *3 points*.

Problem 4. Continuous-Time Convolution. *12 points*.

Convolve the two-sided continuous-time signals

$$x(t) = \cos(\omega_0 t) \text{ and } h(t) = \frac{\sin(\omega_1 t)}{\omega_1 t}$$

Both signals are defined for $-\infty < t < \infty$.

Problem 5. Discrete-Time FIR Filter Design. *12 points*.

Design a discrete-time finite impulse response (FIR) filter that will

- Zero out 0 Hz,
- Zero out all harmonics of 60 Hz, i.e. 60 Hz, 120 Hz, 180 Hz, 240 Hz, 300 Hz, 360 Hz, etc., and
- Pass all other frequencies in the range (-240 Hz, 240 Hz) as much as possible

The FIR filter is linear and time-invariant.

(a) What sampling rate would you use? Why? *3 points*.

(b) How many zeros would the discrete-time FIR filter have? Give formulas for them. Plot them on a pole-zero plot. *6 points*.

(c) Give the formula for the impulse response for the discrete-time FIR filter. Please simplify the formula as much as possible. *3 points*.

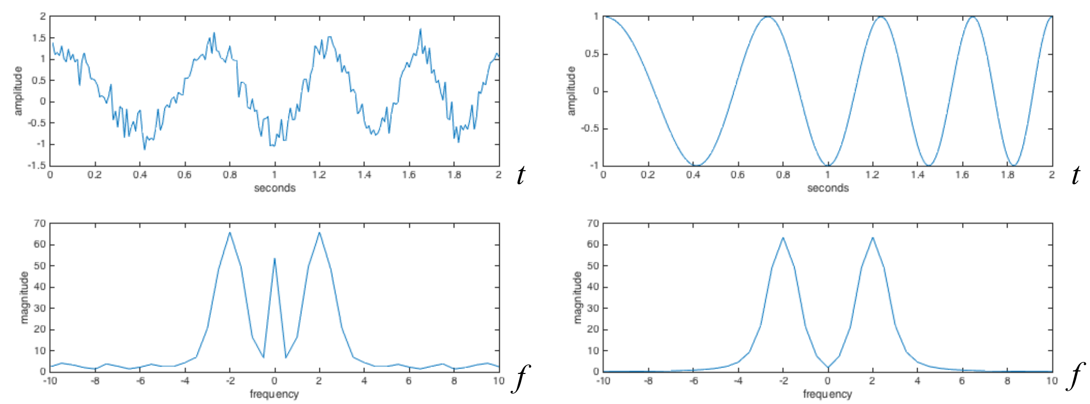
Problem 6. Discrete-Time IIR Filter Design. *12 points.*

A sinusoidal signal of interest has a principal frequency that can vary over time in the range 1-3 Hz.

Using a sampling rate of f_s = 20 Hz, a sinusoidal signal was acquired for 2s and shown below on the left in the upper plot. The lower plot is the magnitude of the signal's frequency content.

The acquired signal has interference and other impairments that reduce the signal quality.

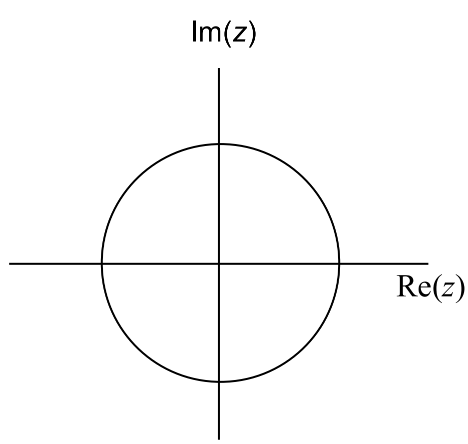
The signal shown below on the right is the sinusoidal signal without the impairments.



Design a second-order infinite impulse response (IIR) filter to filter the acquired signal above on the left to give the sinusoidal signal above on the right

(a) Give the two poles and the two zeros of the second-order IIR filter. *9 points*.

(b) Draw the pole-zero diagram for the second-order IIR filter. *3 points*.



Problem 7. Continuous-Time Feedback System. *12 points.*

Consider a linear time-invariant (LTI) system with input signal $x(t)$ and output signal $y(t)$ that is governed by the following second-order differential equation for $t \geq 0$:

$$y''(t) + 6\, y'(t) + K\, y(t) = x(t)$$

where K is a real-valued constant.

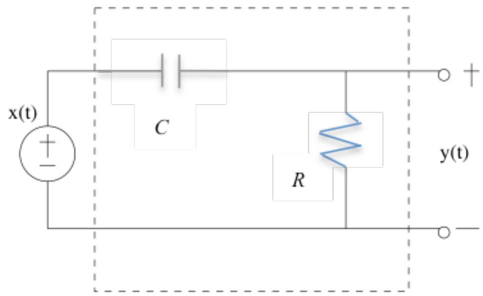
(a) Derive the transfer function $H(s)$ for the system, which will depend on K. *3 points*.

(b) Give the range of values for K for which the system is bounded-input bounded-output (BIBO) stable. *6 points*.

(c) Describe the possible frequency selectivity (lowpass, highpass, bandpass, bandstop, allpass or notch) that the system could exhibit for different values of K for which the system is BIBO stable. *3 points.*

Problem 8. Continuous-Time Circuit Analysis. *9 points*.

Consider the following analog continuous-time circuit with input voltage $x(t)$ and output voltage $y(t)$:



The initial voltage across the capacitor is 0V, and hence, the circuit is a linear time-invariant system.

(a) Using the voltage drop around the loop

$$x(t) - \frac{1}{C}\int_{0^-}^{t} i(t)dt - Ri(t) = 0$$

take the Laplace transform of both sides of the equation to find the relationship between $X(s)$ and $I(s)$. $I(s)$ is the Laplace transform of the current $i(t)$. *2 points*.

(b) Using the formula for the voltage across the resistor

$$y(t) = R\, i(t)$$

take the Laplace transform of both sides and substitute the expression for $I(s)$ obtained in part (a) to obtain the transfer function $H(s)$ in the Laplace domain so that $H(s) = Y(s) / X(s)$. *2 points*.

(c) Find a formula for the frequency response $H(j\omega)$ of the circuit. *2 points*.

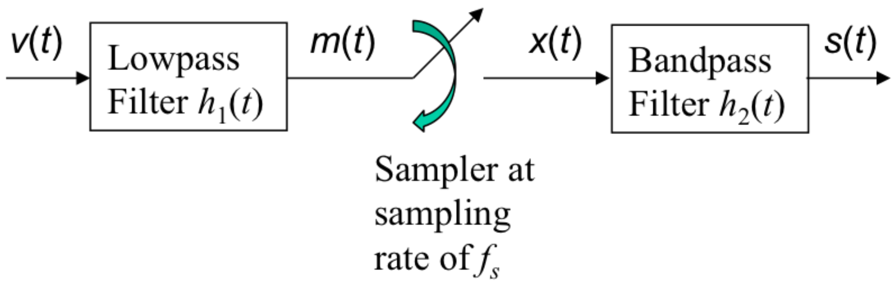
(d) What is the frequency selectivity of the circuit? Lowpass, highpass, bandpass, bandstop, allpass or notch. Why? *3 points*.

Problem 9. Sinusoidal Amplitude Modulation. *12 points.*

Mixing provides an efficient implementation in analog continuous-time circuits for sinusoidal amplitude modulation of the form

$$s(t) = m(t) \cos(2\, \pi\, f_c\, t)$$

where $m(t)$ is the baseband message signal with bandwidth W, and
f_c is the carrier frequency such that $f_c > W$



(a) Assume $h_1(t)$ is an ideal lowpass filter. Give the range of negative and positive frequencies that it passes. *3 points.*

(b) Assume $h_2(t)$ is an ideal bandpass filter. Give the range of negative and positive frequencies that it passes. *3 points*

(c) Draw the magnitude of the Fourier transforms of $m(t)$, $x(t)$, and $s(t)$. You do not need to draw the magnitude of the Fourier transform of $v(t)$. *6 points.*